

Variable Stars – II. Pulsating stars

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1 Introduction

A non varying star is relatively boring as an individual object; we can determine its surface temperature, gravity and element abundances and measure the amount by which it is reddened by interstellar matter, and a number of other parameters. Once a star varies, however, more possibilities appear. We have already seen that binaries – and, in particular, eclipsing binaries – offer the possibility to determine **absolute** values of stellar radius and mass, for example.

When a star pulsates, we have the possibility to find out something about the interiors of the star by matching observations with mathematical models of how stars should pulsate. We have already seen that in at least one (admittedly special) case (V652 Her) we can measure the **rate** of stellar evolution. In addition, pulsation sometimes gives us the ability to use a parameter which is very easy to measure accurately (such as pulsation period) to determine a parameter (luminosity/distance) which might be very difficult to find otherwise.

Fortunately, pulsation occurs all over the HR diagram – this means that it happens in stars at different temperatures, luminosities *and* evolutionary stages, giving us the potential to sample a wide range of stellar types.

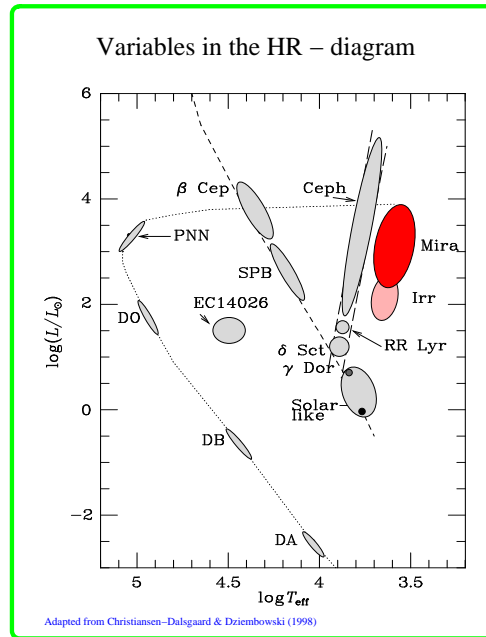


Figure 1: The location of some types of pulsating star in the HR diagram

2 Stellar Pulsation

Two important questions arise:

- HOW does a star pulsate ? That is, what are the natural modes of pulsation ? what does a star look like as it pulsates ? and
- WHY does a star pulsate ? That is, what is the driving mechanism ? or, why do some stars pulsate and not others ?

Let us look at these in turn

2.1 Radial Pulsation

Everything has natural frequencies of pulsation. In the case of a gas sphere (a star), the most obvious mode of pulsation is that **the star remains spherical and simply changes its volume**. The matter all moves along radii of the sphere; the variation is said to be **radial pulsation**.

Of course, radial pulsation – just like a plucked guitar string – can be fundamental, first overtone, second overtone, etc. In fact – and this is very important – **all of these modes of variation can be excited at the same time**.

A star is more analogous to an open-at-one-end organ pipe (see sketch). A node (no movement) exists at the centre of the star (of course – the star is symmetric) and an antinode (maximum movement) exists at the surface.

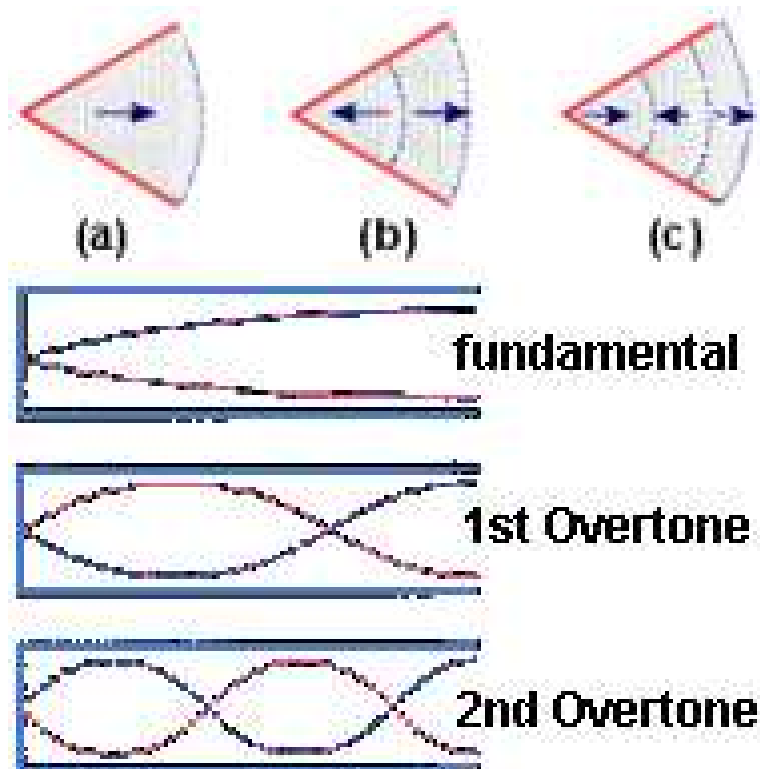


Figure 2: Organ pipe analogy for stellar pulsation.

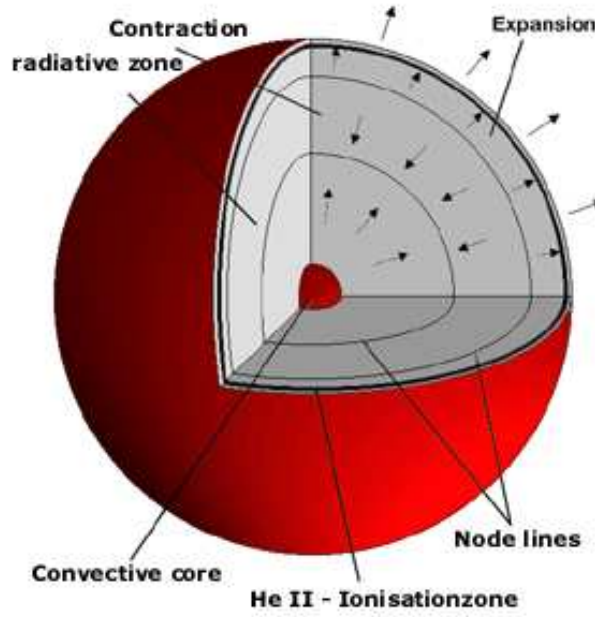


Figure 3: There are three nodes in the radial direction (including the node at the centre of the star) in this schematic pulsation; this is the second overtone of radial pulsation – the **radial order** $n=2$. (where the fundamental radial pulsation would be of order $n=0$).

2.2 Non-radial pulsation

If motion is not along radii, if some parts of the stellar surface move inwards while some move outwards, or **the star changes shape while keeping its volume essentially constant**, we have **non-radial pulsation**.

Non-radial modes are characterised by the number of surface nodes (lines of no movement – as in any vibrating system). The **degree** ℓ indicates the total number of nodes on the surface and **m** the number running through the pole (see figure).

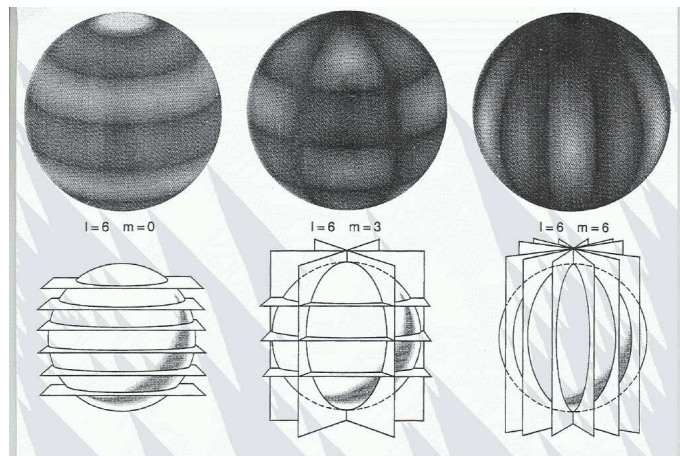


Figure 4: Illustration of non-radial modes – $l=6, m=0$; $l=6, m=3$; $l=6, m=6$.

The next figure illustrates some of the simplest spherical harmonics – the **normal modes** – in which stars can pulsate:

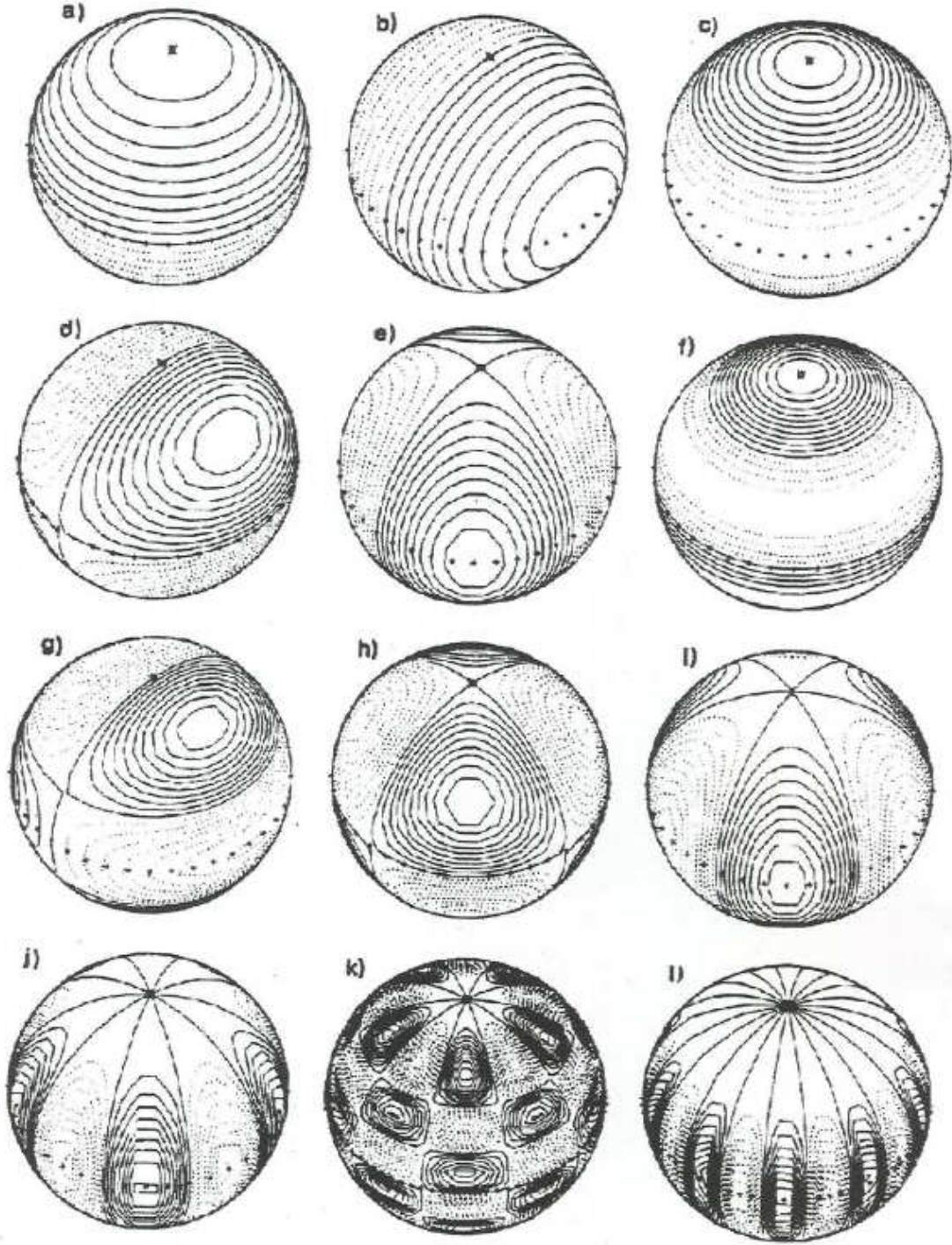


Figure 5: Illustrations of non-radial pulsation modes: (a) $\ell=1, m=0$; (b) $\ell=1, m=1$; (c) $\ell=2, m=0$; (d) $\ell=2, m=1$; (e) $\ell=2, m=2$; etc.

Of course, there is no reason why radial and non-radial effects cannot exist at the same time, so that a mode may be described by three spherical harmonic components ℓ , m and n as in the figure representing one of the solar modes:

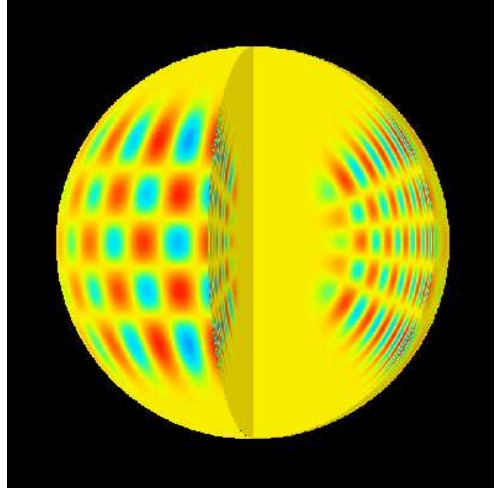


Figure 6: representation of high order (n) and high degree (ℓ) non-radial mode. The different colours represent the surface rising/falling – alternatively cooling/heating.

Equally, there is no reason why many modes cannot all be excited at the same time – just as with a plucked string, the fundamental and many overtones might be present. In the Sun, modes have been identified from order 0 to over 1500 !

Because we can see the Sun as a well-resolved disk, it has been possible to identify many thousands ($> 100\,000$) of pulsation modes. For other stars, which we see as point sources, the higher degree modes (greater than $\ell = 3$ or 4) generally are not seen, because then the surface of the star is divided into a number of somewhat hotter and somewhat cooler “patches” and these tend to average out to the mean temperature over the pulsation cycle(s).



Figure 7: Similar to the above figure but with surface displacements shown in *highly* exaggerated form.

2.3 Driving pulsation

We have seen that potentially a star might pulsate in many modes. But, as with any vibration, there will be a natural tendency for it to be damped unless there is some repeated or continuous driving mechanism.

2.3.1 The ϵ mechanism

The energy generation rate in stellar cores is usually denoted by ϵ , (the Greek character “epsilon”). Since energy generation is dependent on high powers of the temperature, it might be supposed that small variations, even statistical fluctuations, could lead to variations in energy generation rates which might be self-sustaining.

From mathematical models, however, it seems that this is only likely to occur in fully-convective stars – such as the coolest M dwarfs – and in the most massive stars – perhaps with $M > 60M_{\odot}$. Such a mechanism is unlikely to influence the great majority of observed pulsating stars.

As an analogue, recall that radial pulsation modes have a node at the centre of the star, which is where the nuclear energy generation occurs. Applying a driving force at a node is the same as trying to pluck a guitar string at one end. You can get a sound, but it’s very difficult compared to plucking the string near the middle.

2.3.2 The κ mechanism (and γ mechanism)

The opacity of a star – the factor which determines how radiation diffuses from the interior outwards – is usually represented by κ , (the Greek “kappa”). Opacity depends on a number of parameters, the atoms involved, density, the wavelength of the radiation and so on, but a key factor in understanding pulsation is the ionization of matter. Ionised matter contains free electrons and – at the temperatures inside a star – electron scattering and free-free absorption will dominate the opacity. This leads to the mechanism sometimes called the “Eddington valve” but more usually nowadays, the “ κ -mechanism”.

Consider a spherically symmetric star. At some depth into the star there will be a zone, above which hydrogen is neutral and below which it is completely ionized.

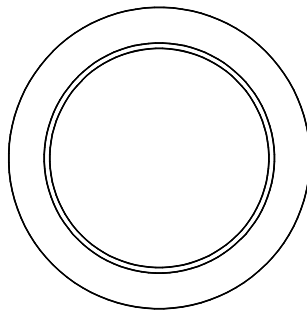


Figure 8: Schematic (not to scale) of a partial ionization zone – a spherical shell – inside a star.

In this zone, some hydrogen atoms will be ionized, others neutral, and a small change in temperature might substantially change the ratio of neutral to ionized atoms. It is referred to as a

partial ionization zone – accurate, if unimaginative.

At some depth, there will be a zone where helium is singly ionized and, deeper, a zone where it is doubly ionized. Similarly, there will be various zones of different ionization for all the elements, though we know that in most stars H and He dominate the elements.

Outside partial ionization zones, if a star is compressed, it heats up, the radiation flow increases and the opacity actually decreases (it scales approximately as $\kappa = \rho/T^{3.5}$ – Kramers Law), so for a given shell of material, more energy is lost at the upper level than is received at the lower. This **radiative damping** very quickly damps out the pulsation

Now consider what happens in a partial ionization zone if a star is pulsating:

- as the star is compressed, the energy which would normally heat the zone mostly goes into increasing the ionization. This increases the opacity of the zone, trapping radiation more efficiently, and resulting in outward pressure. This damming of the radiation is sometimes referred to as the γ mechanism;
- the zone is then driven outwards, cooling as it rises, which also increases the opacity and outward pressure;
- further cooling of the zone results in recombination of the ionized material, a sudden decrease in the opacity, a decrease in outward pressure, and the zone drops back.

Thus, through a pulsation cycle, the partial ionization zone acts with any pulsation movement. In this case the star is **unstable to pulsation** and any small variation, will be reinforced – the pulsation will grow until the energy input by the κ -mechanism reaches a limit. The star will then be pulsating with a stable period and amplitude.

Note that the partial ionization zones can be near the surface of the star, so that they act near an antinode where driving should be easiest.

It's not quite as simple as that. If the zone is too deep in the star, it cannot drive against the overlying layers and if the zone is too high, it has essentially nothing above it to drive. Placement of the zone is thus fairly critical in determining whether pulsation occurs – and explains why there are so-called “instability strips” in the HR diagram. These are areas where the stellar temperature is such that the driving zone is well located.

The hydrogen and neutral helium partial ionization zones occur at roughly the same depth ($T \sim 15000\text{K}$). Clearly, these would not work for O stars where such zones would be on the surface of the star (if they existed; we can see that hydrogen is mostly ionized in an O star atmosphere).

Even for the much cooler classical Cepheid stars (δ Cepheids), the driving zone is believed to be the second ionization of helium ($T \sim 40000\text{K}$). For hot stars, like the β CMa pulsators, and the rapidly pulsating sdB stars, the driving is likely to be by “iron-peak” elements at perhaps 200 000K.

2.3.3 Stochastic processes

In the Sun, it is believed that the observed **very** small variations (typically at the micromagnitude level rather than the $>$ millimag level which is usually all we can observe in stars) are maintained by stochastic noise generated by convection near the surface. Such variations are extremely difficult to detect in other stars, though some progress has been made.

2.4 The Baade-Wesselink method

To give an illustration of how we can obtain fundamental information from a pulsating star (which we wouldn't be able to get from a star in equilibrium) we look at the Baade-Wesselink method for determining the radius of a pulsating star. Unsurprisingly, this method was devised by Walter Baade in the 1920's and developed by Adriaan Wesselink in the 1940's.

It goes something like this:

The equation which defines the **effective temperature** (I write T for T_{eff}) of a star is:

$$L = 4 \pi R^2 \sigma T^4$$

so:

$$-2.5 \log L = 5 \log R - 10 \log T + \text{constant}$$

or:

$$M_{bol} = -5 \log R - 10 \log T + \text{constant}$$

If the star pulsates, we will get a radius, temperature and luminosity change between two times t_1 and t_2 , and we can write:

$$m_{bol,1} - m_{bol,2} = M_{bol,1} - M_{bol,2} = 5 \log \frac{R_1 + \Delta R}{R_1} + 10 \log \frac{T_2}{T_1}$$

where the radius change ΔR is given by integrating the velocity curve:

$$\Delta R = -p \int_{t_1}^{t_2} v(t) dt$$

where the coefficient p corrects for mainly for projection effects but also for limb darkening (and is about 24/17).

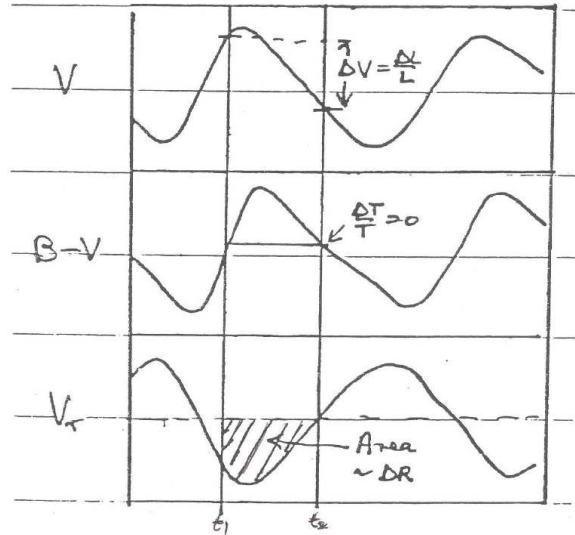


Figure 9: Sketch of Baade-Wesselink quantities.

The difference $m_1 - m_2$ can be taken from the V light curve. Strictly speaking, of course, bolometric corrections would be required to both magnitudes, but these – and the effective temperatures – can be found from the $(B - V)$ colour curve for example.

Alternatively, t_1 and t_2 could be chosen so that the colours were the same and then the bolometric corrections would cancel and the temperature term disappears (see the figure).

Since we can measure everything but R_1 from the light and velocity curves, we can find the stellar radius, in absolute terms, as a function of time (or pulsation phase).

Note that this method would not work for non-radial pulsation because the simple expression for ΔR does not allow for non-radial components

2.5 Other considerations

I list here some terms which we don't have time to look at in any detail, but which you should have at least heard !

- **Radial pulsations** are a special subset (with $\ell = 0$) of the more general **non-radial pulsations** or non-radial **pressure modes** (often written **p-modes**) – for which the restoring force is pressure.
- Non-radial pulsators can also pulsate in **gravity modes** or **g-modes**, where gravity – actually **buoyancy** – is the restoring force. g-modes have been described as material “sloshing back and forth” and g-mode periods will be different from p-mode periods for the same spherical harmonic.
- In theoretical modelling of stellar pulsation, the simplest case is to assume that the pulsations are **adiabatic** (no energy lost from the mechanism) and **linear** – this means that quadratic and higher terms can be “safely” ignored. The latter requires that the pulsations have rather small amplitude – then the variations are essentially sinusoidal. You can see this is not true (for example) for Cepheids, where the amplitudes are neither small nor sinusoidal.
- In more complicated models, **non-adiabatic** effects can be allowed for – then one has **linear, non-adiabatic** models. Most difficult is to try to allow for non-linear effects (large amplitude pulsations) and then one has **non-linear, non-adiabatic** models.
- Even in the most complex models, there are many effects which cannot be easily included – but which we know will have some effect. Perhaps the most obvious of these are **convection** which could significantly alter – even destroy – pulsations; **magnetic fields**; **differential rotation** of the star; and so on.

3 Some examples of types of pulsating star

3.1 Classical Cepheids = Type I Cepheids = δ Cephei stars

Cepheids are variable supergiant stars with similar temperatures to the Sun. They are named after the prototype δ Cephei and are also called δ Cepheids or type I Cepheids, for reasons which will become evident.

The first recorded discovery of the variation in δ Cephei was by John Goodricke in 1784. (Goodricke was perhaps more famous for his explanation of the eclipsing binary nature of Algol; he was elected a Fellow of the Royal Society but died of pneumonia at the age of only 21).

For nearly a hundred years, Cepheids have been believed to be pulsating stars. They are radial pulsators with maximum velocity of approach (of the moving surface of the star) near light maximum and bluest colour (recall that $L = 4\pi R^2 T^4$).

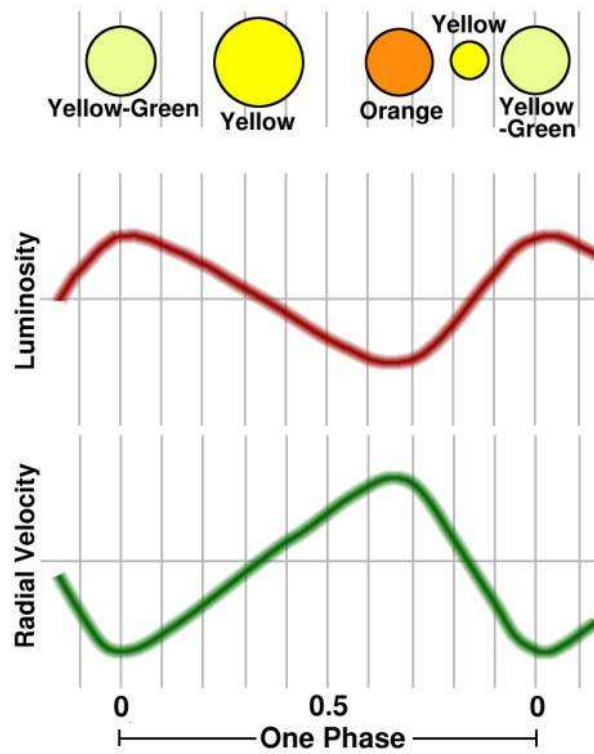


Figure 10: Schematic of luminosity, colour and velocity variations for a classical Cepheid pulsator – a radial, fundamental mode pulsator.

As noted above, Cepheids have similar temperatures to the Sun (something like 5000 to 7000K; spectral types from early F to K) and range in luminosity from about $500L_{\odot}$ to about $30000L_{\odot}$ which makes them detectable at great distances. Their periods are typically from about 1 to 50 days.

For individual stars, light amplitude variations are typically 0.5 to 2 magnitudes (again making them relatively easy to detect), and they have velocity amplitudes $\sim 30 - 60$ km/s. The next two figures show δ Cephei itself, as an example.

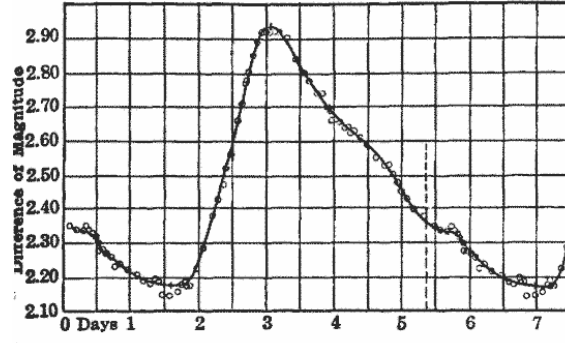


Figure 11: Actual light curve for δ Cephei itself.

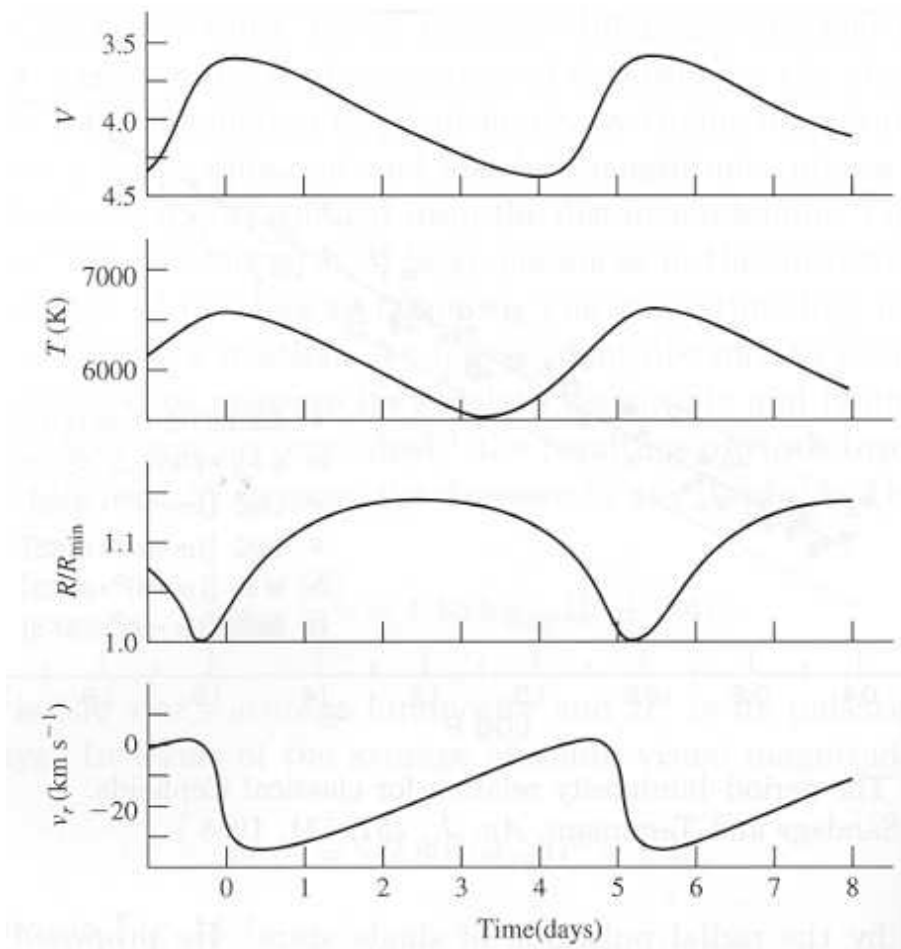


Figure 14.5 Observed pulsation properties of δ Cephei.

Figure 12: Variation of various parameters for δ Cephei.

During the period 1908 – 1912, Henrietta Leavitt, a “computer” at Harvard College Observatory recognised that there was a relationship between the period and the luminosity of the Cepheid variables in the Small Magellanic Cloud.



Figure 13: Henrietta Leavitt (1868 – 1921).

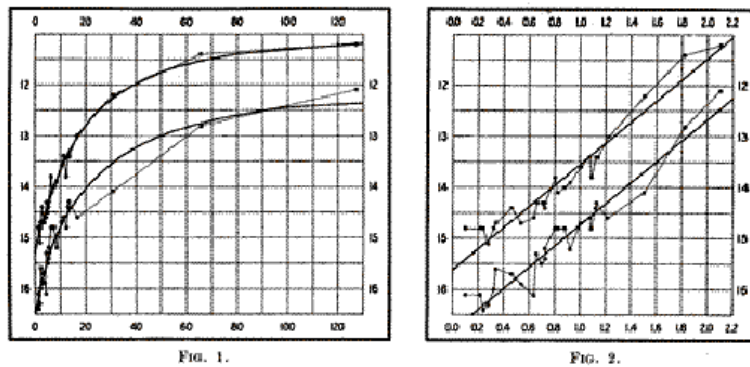


Figure 14: Leavitt’s Period-Luminosity relationship for the maximum and minimum brightnesses of 25 SMC Cepheids. Note that this is not an absolute calibration as the true zero point of the plots was not known.

Leavitt was able to derive this relationship because the size of the SMC is small compared to its distance from us so, although not strictly true, it is not unreasonable to assume that all the stars in the SMC are at the same distance. Had she lived longer (she died of cancer at age 53) she might well have earned the Nobel prize for her work.

The importance of this relationship derives from the facts that:

- with a zero point for the relationship, we can use the measured pulsation period (easy to measure accurately) to obtain the luminosity and hence distance of Cepheids;
- since Cepheids are supergiants, they will be visible at great distances – even extra-galactic distances – they thus become a vital “stepping stone” in the effort to calibrate the scale of the Universe;
- since they have relatively large amplitudes ($\sim 0.5 - 2$ mag) and distinctive light curves, they should be easy to identify, even if apparently quite faint (i.e. very distant) and in the crowded fields of galaxies.



Figure 15: The Small Magellanic Cloud (SMC) - a satellite galaxy of our own Galaxy.

Because of this importance, a great deal of work has gone into the accurate calibration of the zero point of the period-luminosity relation. Early efforts turned out to be wrong for various reasons (unknown at the time), but calibrations steadily improved and Cepheid P/L distances were used first in studies of our own Galaxy (e.g. the rotation curve) during the middle of the 20th century, and later in determining the extra-galactic distance scale.

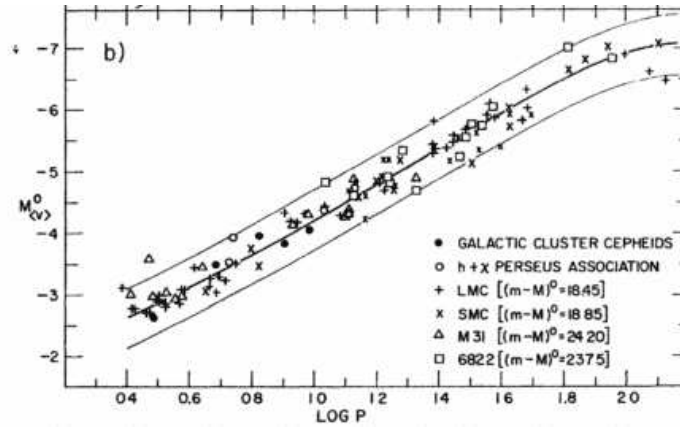


Figure 16: Period-luminosity relation by Sandage & Tamman (1968). Note that they used Cepheids in galactic clusters and associations (with distances determined by main-sequence fitting) and also nearby Galaxies.

More recently, Feast & Catchpole (1997) – both previously at the SAAO – used *Hipparcos* satellite parallax measurements to derive:

$$\langle M_v \rangle = -2.8 \log P - 1.43$$

Where $\langle M_v \rangle$ is the mean magnitude of the variable. Other modern measurements agree well with these values for both slope and zero-point. As a rough illustration, a Cepheid with a period near 2 days will have $L_* \sim 300 L_\odot$, whereas one with $P \sim 50$ days will have $L_* \sim 30000 L_\odot$.

The next figure illustrates the kind of data obtained for extra-galactic Cepheids - *Hubble Space Telescope* frames of the galaxy M100

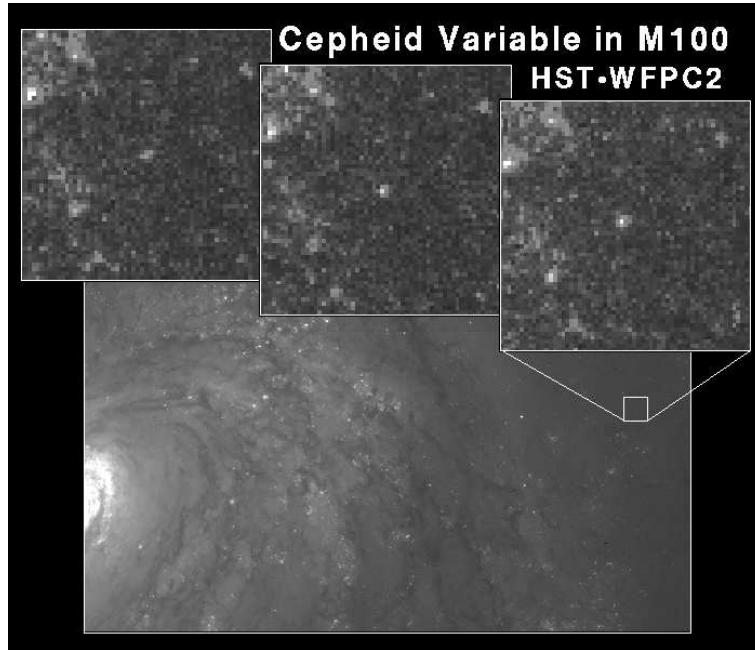


Figure 17: HST observations of a Cepheid in the galaxy M100.

In recent years, multi-colour photometry has shown that the light curves have systematically decreasing amplitude with wavelength.

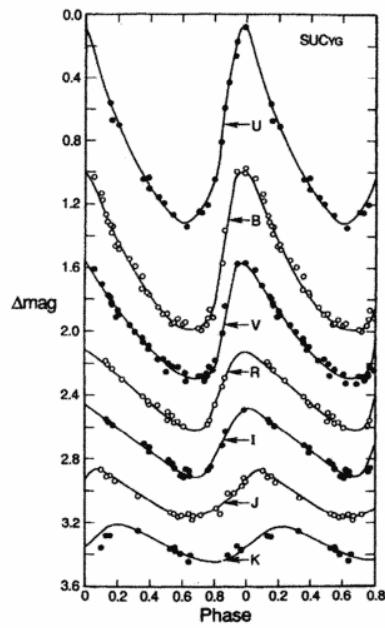


Figure 18: Multi-colour observations of a Cepheid variable.

This has a natural advantage that measurements made in the infrared (JHKL) will yield a more accurate mean magnitude for a Cepheid (because the variation is so small). On the other hand, it's easier to find them and to determine the period accurately using bluer passbands.

A bonus of using infrared as well as optical data is that the P/L relation looks much “tighter” at infrared wavelengths (see figure) so that, potentially, one should get more accurate results for luminosity (and therefore distance) by using, say, a K magnitude. A snag is that it's much harder to measure faint, extragalactic stars in the infrared.

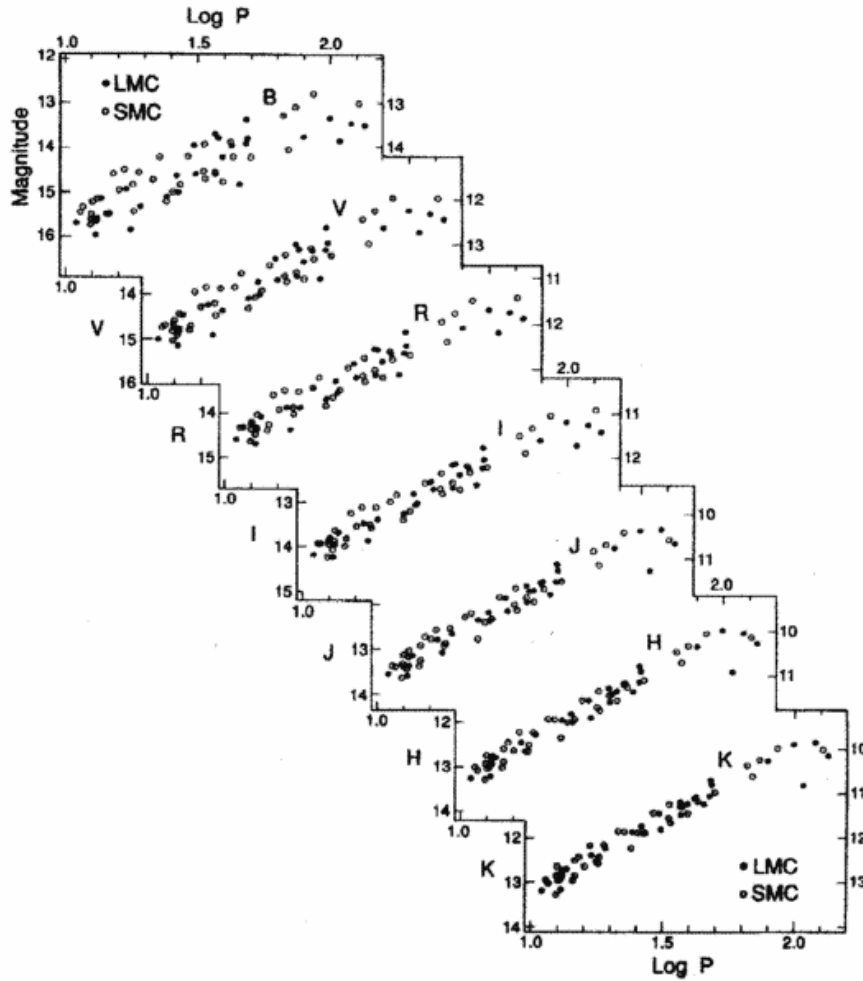


Figure 19:

It is not certain why the scatter in P/L calibrations should be less at redder wavelengths – certainly the effects of interstellar reddening are less at the longer wavelengths, and it is likely that errors in correcting for reddening contribute more scatter at bluer wavelengths. The scatter at the reddest passbands shown might well reflect the real thickness of the instability strip.

To find out more about the history of the Cepheid P/L calibration and the scale of the Universe, see www.institute-of-brilliant-failures.com.

3.2 W Vir stars = Type II Cepheids – and galactic populations

One of the reasons the early P/L relations for Cepheids went wrong was because the “classical” Cepheids found in the Magellanic Clouds and our Galaxy were identified with stars with very similar light curves found in globular clusters – the so called “long-period” cluster variables – now called type II Cepheids, or **W Virginis (W Vir)** stars.

During the second World War, Walter Baade took advantage of dark skies and the large amount of available telescope time at Mt Wilson, to take “deep” plates of the galaxies M31, M32 and NGC 205. He was able to resolve stars in these galaxies and realised that there appeared to be two “populations” of stars – one which corresponded to young stars, open clusters and galactic spiral arms (which he called population I) and another with a rather different HR diagram which corresponded to globular clusters and galactic “bulges” (population II).

The terms population I and II persist, though the concept is very much a simplification. It is believed that when the Galaxy (and other galaxies) formed, it did so from “primeval” material - composed of hydrogen, helium and very little else. As stars formed and evolved, they dumped nuclear processed material back into the interstellar medium (at giant and supernova stages, for example) thus enriching the interstellar material with “metals” – elements heavier than helium. The metal-poor stars of globular clusters and the galactic bulge form “population II” whilst the relatively metal-rich stars of the galactic disk are “population I”.

We now believe this to be an oversimplified (though *essentially* correct) picture. It is common nowadays to recognise a series of Galactic populations, something like:

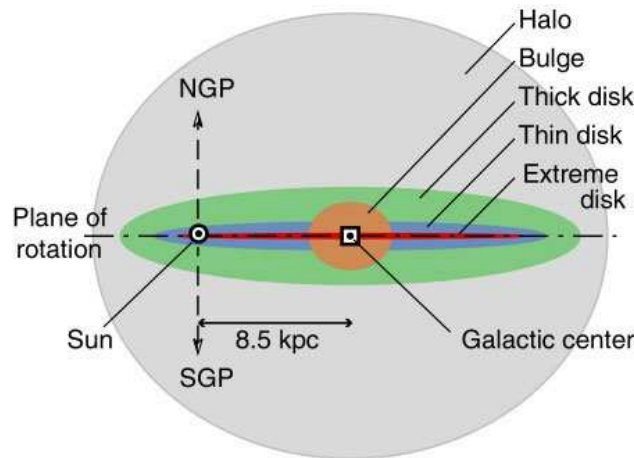


Figure 20: Schematic of various Galactic “populations”.

These might be described as:

- **thin disk** – population I – the youngest stars, massive OB stars, classical Cepheids, etc.
- **thick disk** – intermediate population II – somewhat less metal-rich stars.
- **bulge** – population II – evolved stars; a range of metallicities.
- **halo** – extreme population II – lowest metallicity stars, globular clusters, etc.

Of course, this is still a simplification; there would have been a gradual increase in metallicity with time, though there might well have been “bursts” of star formation as well.

Important also to realise that these populations intersect to some extent. For example, the Sun – at 5 billion years old, not the youngest of stars – is very close to the plane of the Galaxy and orbits in (probably) a near circular/planar Galactic orbit. Near the Sun can be found very low metallicity “high velocity” stars. These are halo stars, often travelling at low speed in high-inclination, high-eccentricity orbits, which the Sun is passing at high velocity in its Galactic disk orbit.

If we now look at a more modern representation of the P/L relations for type I and type II Cepheids, we can see that assuming an absolute magnitude for type I – based on the luminosities of globular cluster stars (pop II) would result in an error in the absolute magnitude of about 1.5 mag (differences ranging from about 0.7 to around 2.0 mag).

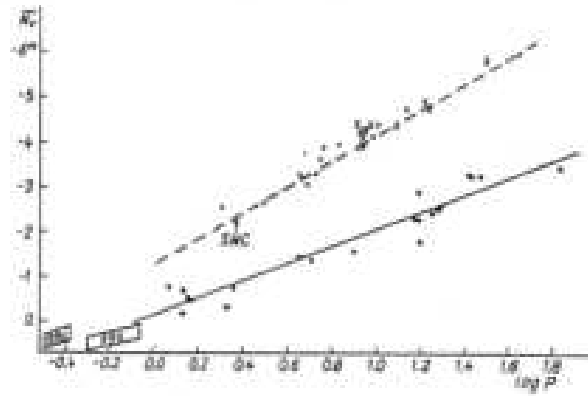


Figure 21: Period/Luminosity relations for type I (upper) and type II Cepheids, and RR Lyrae stars.

When this error was realised – following the work of Baade – it meant that the magnitude error of ~ 1.5 mag corresponds to a brightness error of about 4 and thus a distance error of 2. The Universe was suddenly twice as big as had been previously imagined !

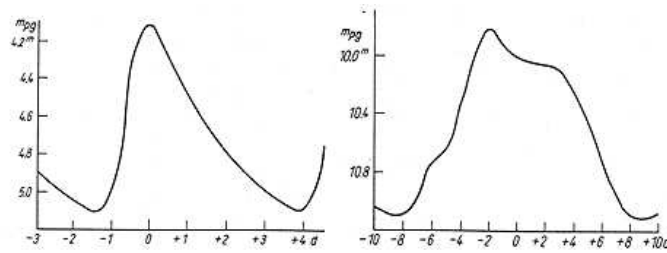


Figure 22: Comparison of light curves for δ Cephei (left) and W Vir (right).

It is now recognised that there are small differences between type I and type II Cepheids, in light curve shape (see figure), amplitude and spectral appearance (metallicity).

Typical W Vir periods are from a few to ~ 35 days and the General Catalogue of Variable Stars (GCVS) classes type II Cepheids as:

- **CWA** – variables with periods > 8 days (W Vir stars)
- **CWB** – variables with periods < 8 days (BL Her stars)

although another scheme (Wallerstein, 2002) is:

- **BL Her** – variables with periods < 5 days
- **W Vir** – variables with periods up to 20 days
- **RV Tau** – variables with periods > 20 days

Stellar evolutionary tracks indicate what we now believe about the Cepheids. Type I Cepheids are massive ($M_* \sim 3 - 9M_\odot$) stars which are young but have evolved away from the main sequence. Type II Cepheids are old, low mass stars (M_* probably $\sim 1M_\odot$) which have evolved away from the main sequence, up the giant branch (as hydrogen fusion ends), down to the horizontal-branch (after helium fusion switches on), back up the asymptotic giant branch (as helium fusion ends), but are experiencing helium “flashes” as helium burning briefly switches on again. This shifts the star to higher temperature and over to the instability strip, where the type II Cepheid variations occur.

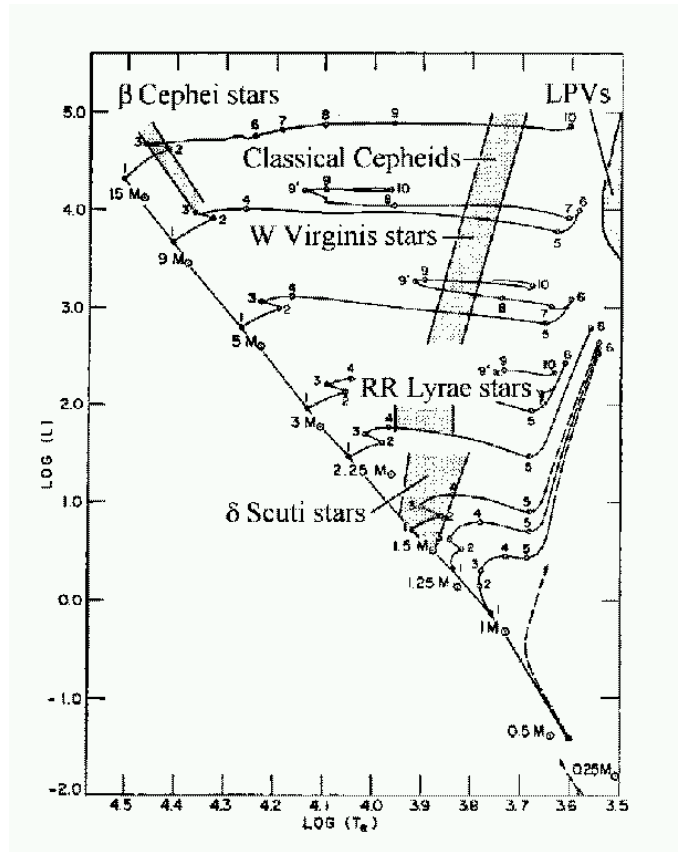


Figure 23: Stellar evolutionary tracks and instability strips.

As can be seen in the figure, both types of Cepheid and the RR Lyrae pulsators (horizontal-branch stars) are of similar temperature and are likely driven by the same mechanism (the second helium ionisation zone). The β Cephei stars (at the upper left of the HR diagram) are clearly much hotter and are believed to be driven by iron-peak element ionisation at much higher temperature.

3.3 Overtone Cepheids and double-mode pulsators

The description of type I and II Cepheids presented above is that normally given in text books, which usually show a picture somewhat similar to this:

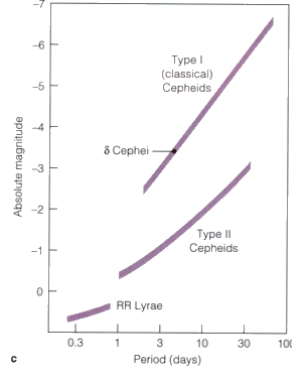


Figure 24: Schematic of type I and type II Cepheids (and RR Lyrae stars).

However, as usual, the situation is more complex. It turns out that whilst many Cepheids pulsate in the fundamental mode, some are first overtone (radial) pulsators. These are apparently uncommon in the Galaxy, but a massive study by Cecilia Payne-Gaposchkin and Sergei Gaposchkin (*Vistas in Astronomy, Vol. 8, 191, 1966*) of Magellanic Cloud Cepheids revealed that about 10% appeared to be on a different P/L relation. These are believed to be first overtone pulsators.

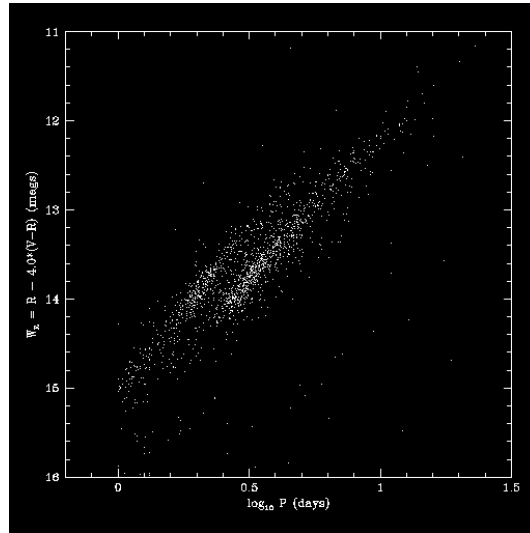


Figure 25: Results from the MACHO survey (a search for **M**ASSIVE **C**OMPACT **H**ALO **O**BJECTS). As a by-product many variables in the LMC were observed. The plot shows a P/L relation for ~ 1500 variables. The lower of the two obvious sequences is formed of fundamental mode Cepheids; the upper band is formed of first overtone pulsators. A few type II Cepheids are seen in the lower right of the figure, amongst other variables. The web page www.macho.mcmaster.ca/Demos/Cepheids/WebPL.html allows you to look at light curves from the above figure.

The story doesn't end there – some Cepheids show *both* fundamental and first harmonic pulsations or first and second harmonic pulsations. These are known as **double-mode Cepheids** and their light curves are naturally much more complex.

3.4 RR Lyrae stars

RR Lyrae stars are also radial pulsators. They are **horizontal-branch** stars, having evolved from the **main sequence** to the **red giant** stage, following the end of Hydrogen fusion. The onset of Helium fusion (to Carbon and Oxygen) puts the star on the horizontal branch at temperatures around 7000K (\sim A type) and luminosities $\sim 50 L_{\odot}$. Typical pulsation periods are about 0.2 to 1.2 day, with amplitudes from a few tenths of a magnitude to about 2 magnitudes.

RR Lyrae stars are common in globular clusters and were originally called “cluster variables” though the term is less popular nowadays. They can be detected in nearby galaxies – and are useful distance indicators because of their reasonably well-determined mean magnitude.

Recall that the term “horizontal-branch” comes from the globular cluster colour-magnitude (HR) diagram. The gap in the horizontal branch is a region where stars are evolving rapidly; they either stay on the red side or evolve fairly quickly to the blue side. RR Lyrae variation is seen in the gap.

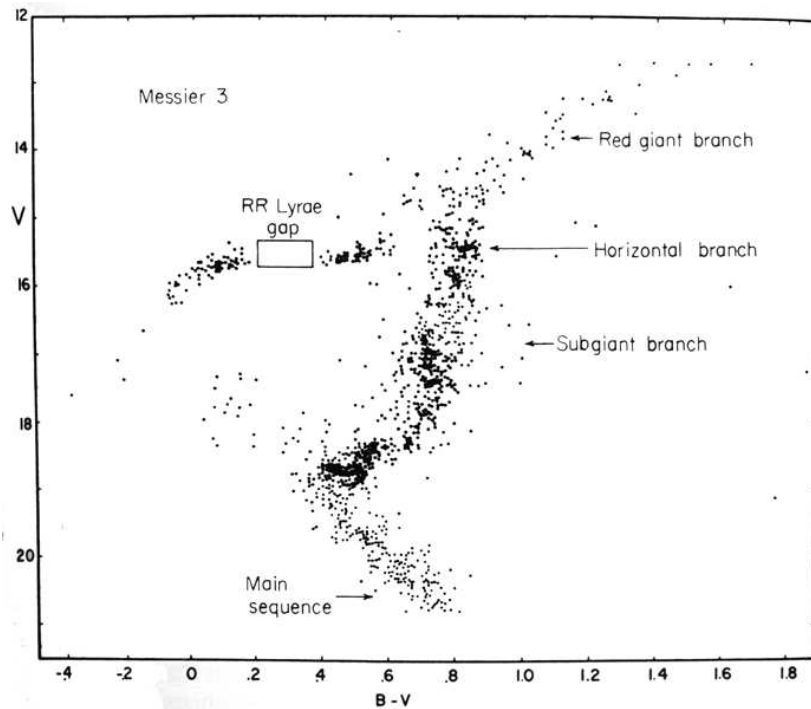


Figure 26: Colour-magnitude diagram of a globular cluster.

There are several sub-types of RR Lyrae star, classified by the appearance of their light curves:

- **RRa**: Asymmetric light curves with a fast rise and slower decline. Radial fundamental mode pulsators.
- **RRb**: Same as RRa type, but with smaller amplitude (~ 0.6 mag). Radial fundamental mode pulsators.
- **RRab**: Sometimes RRa and RRb are lumped together as RRab.
- **RRc**: Nearly sinusoidal light curves with amplitudes ~ 0.5 mag. Radial first overtone pulsators.

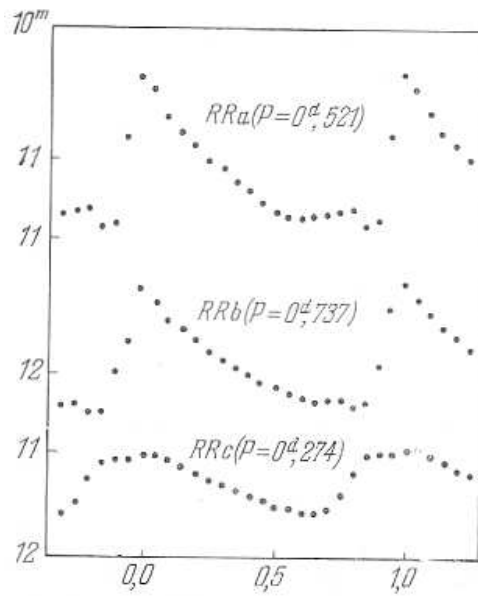


Figure 27:

- **RRd: “Double mode”** RR Lyraes, pulsating in the fundamental and first overtone radial modes.

Interestingly, some RR Lyrae stars show a modulation of the amplitude and shape of their light curves while the pulsation period remains constant. This is called the **Blazhko effect** and is illustrated in the figure.

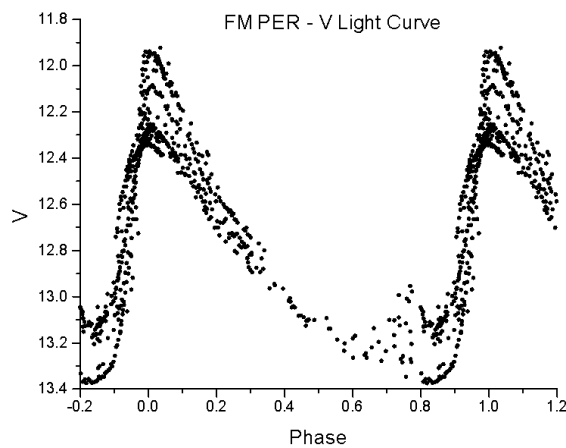


Figure 28:

For a cool animation see:

www.physics.mcmaster.ca/research/Astrophysics/Astrophysics.html

The cause of the Blazhko effect is still unknown.

3.5 δ Scuti stars

δ Scuti stars pulsate in the classical Cepheid “instability strip”.

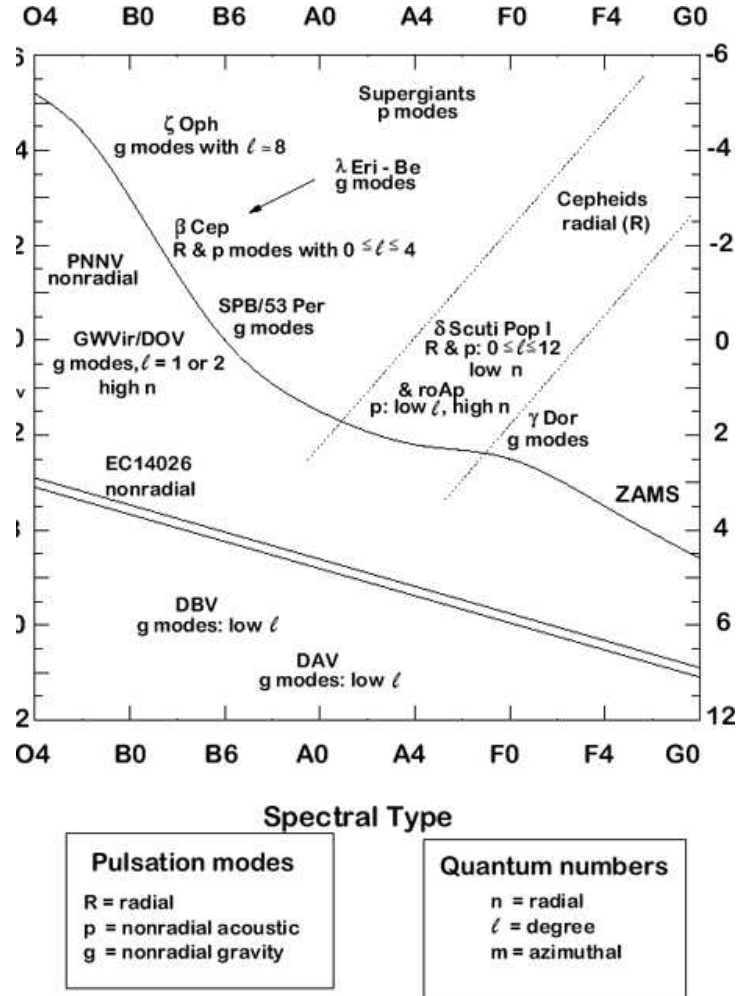


Figure 29: HR diagram showing a number of different pulsating star types.

They are early A to F type stars and of luminosity classes V – III, so main-sequence or slightly evolved stars with masses greater than about $2 M_{\odot}$. Their periods typically lie in the range 30 minutes to about 8 hours and their amplitudes are less than a magnitude.

The δ Scuti stars were originally thought to be related to the classical Cepheids – and indeed were originally called “**dwarf Cepheids**”.

The early nomenclature was somewhat confused – the term “**AI Vel stars**” was used for a while to refer to δ Scuti stars with amplitudes > 0.3 mag; these are now generally referred to as “**High amplitude δ Scuti stars**” (or HADS) and the smaller amplitude varieties are just called δ Scuti (or DS) stars.

Another distinction is made – the metal-weak DS stars are usually called “**SX Phe**” stars. These are population II or old disk population stars. They are something of a puzzle, as they should be far too old to still be near the main sequence. It is possible they are the result of merged binary stars.

The high-amplitude DS stars are pulsating in radial fundamental or first overtone modes; their light curves are often simple.

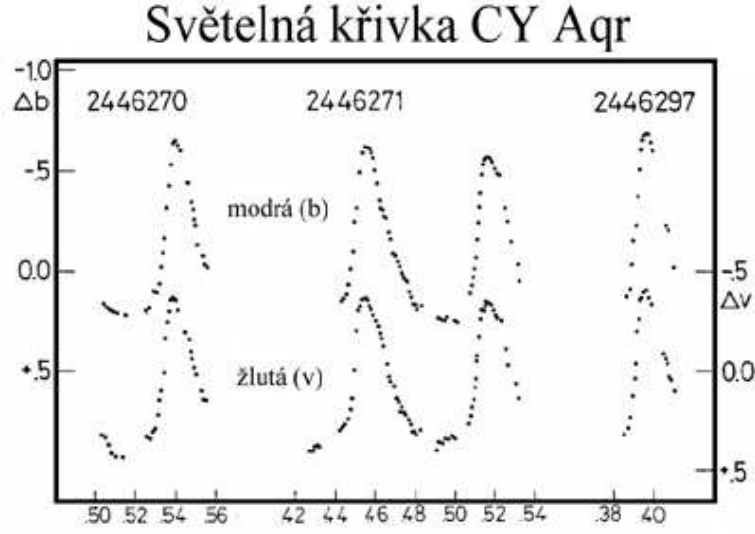


Figure 30: Light curve for CY Aqr (HADS).

As smaller amplitude DS stars were observed – and observed more carefully – it became obvious that these had several modes of pulsation (more than 20 detected in some stars) and that they are pulsating in non-radial p-modes (and perhaps g-modes in some cases).

The next figure shows about half of the data from a 1995 multi-site campaign on FG Vir. The full data set revealed 24 frequencies.

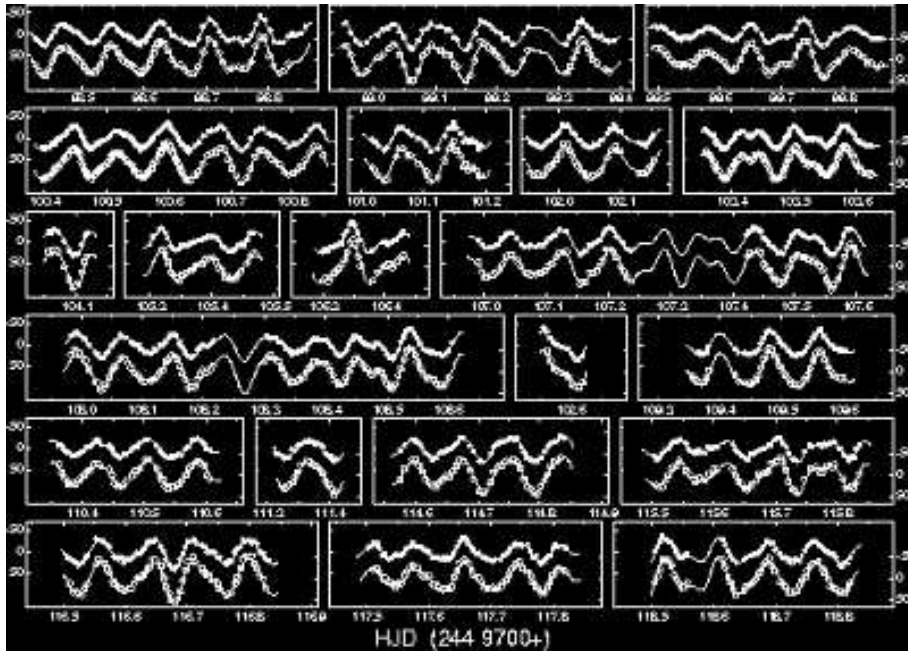


Figure 31: Data from the 1995 campaign on FG Vir (Breger et al, 1999).

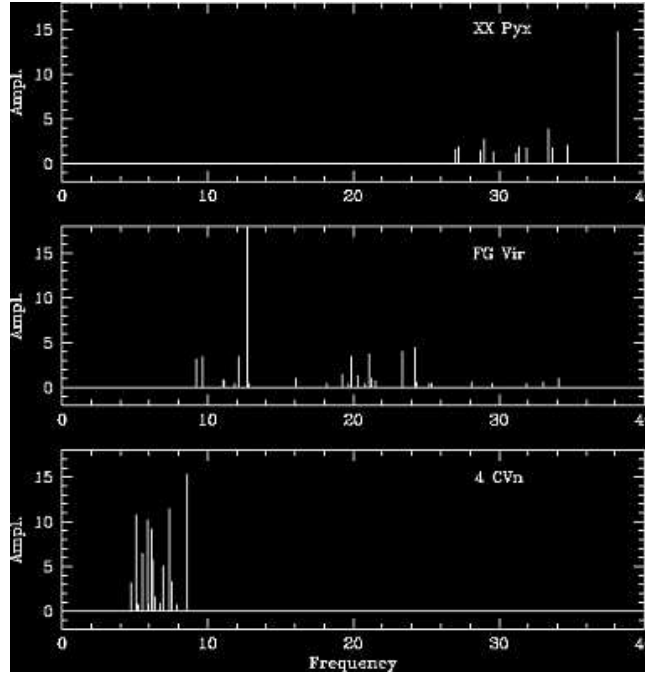


Figure 32: Some representations of the frequencies/amplitudes found for some well studied δ Scuti stars.

At present, it appears that maybe 30% of A and F stars actually show δ Scuti variations. Of course, it could easily be a higher percentage – it could well be that the more closely we look, the more we will find even lower amplitude variations ...

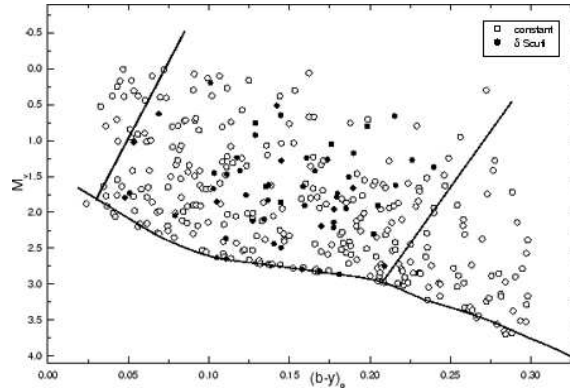


Figure 33:

A Period-Luminosity-Colour relation exists for the DS stars. Breger & Bregman (1975) give:

$$M_V = -3.052 \log P + 8.456 (b-y)_0 - 3.121$$

so they can also be used as distance indicators.

A good site to start looking at δ Scuti stars is:

www.deltascuti.net/DeltaScutiWeb/index1.html

3.6 γ Doradus stars

I briefly mention γ Dor stars because the prototype was discovered to be variable at the SAAO (Cousins & Warren) but only after several years did Cousins realise that the star was multi-periodic, with periods of 0.7570 and 0.7334. The longer periods – near a day – and small amplitudes (< 0.1 mag) mean that these stars were more difficult to recognise.

They are close to the δ Scuti stars in the HR diagram –

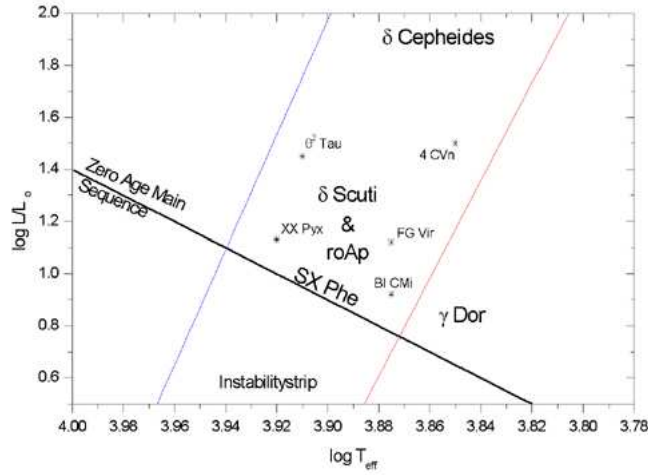


Figure 34: Location of γ Dor stars in the HR diagram.

but are generally cooler and are pulsating with periods of the order of 1 – 2 days and so appear to be non-radial **g-mode** pulsators. This makes them very interesting, as the g-modes potentially allow sampling of the deep interior of the star.

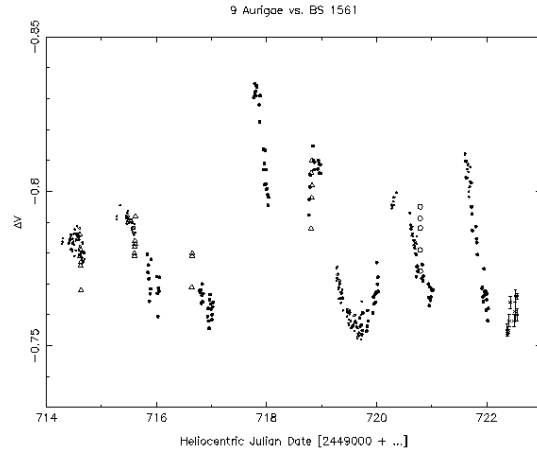


Figure 35:

There are one or two stars which appear to exhibit δ Scuti AND γ Dor pulsations at the same time. Potentially, this gives the ability to sample the centre of the star and the outer layers.

3.7 Pulsating white dwarf stars (DOV, DBV, DAV)

Recall that white dwarf stars can originate from post-horizontal-branch evolution (sdB stars which are not massive enough to become asymptotic giant branch stars) or post-asymptotic-giant-branch stars which shed a planetary nebula before becoming very hot white dwarfs.

Once a star reaches the white dwarf stage, it is mostly electron degenerate and so cools at essentially constant radius – because electron degeneracy pressure is independent of temperature – and because the white dwarf has no internal source of (nuclear) energy.

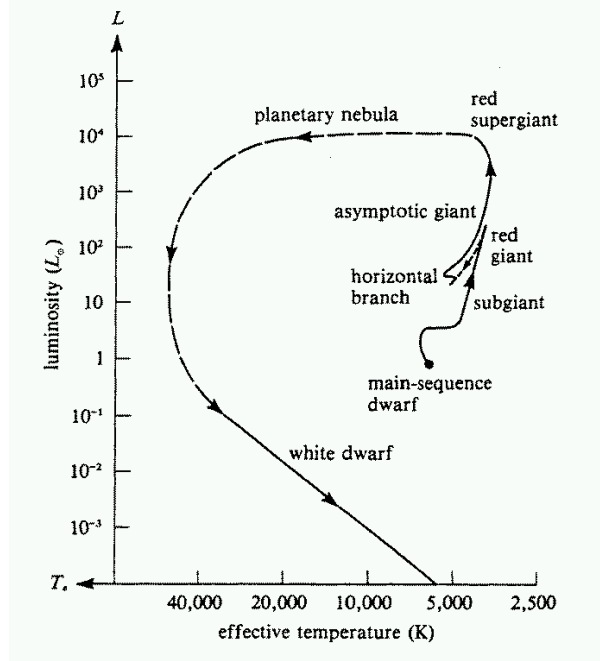


Figure 36: Schematic for the evolution of a solar-type star through giant and asymptotic giant branch stages to the cooling white dwarf.

As white dwarf stars cool, they pass through areas in the HR diagram where pulsation can occur (i.e. where the structure of the star allows pulsation).

It turns out that theory predicts that the fundamental radial mode for a white dwarf will typically have a period of ~ 1 second, and that higher order p-modes will have even shorter periods. Since the pulsations we observe in white dwarf stars typically have periods $\sim 100 - 1000$ seconds, these are believed to be **non-radial g-mode pulsations**.

3.7.1 White dwarf classification

The current white dwarf classification scheme uses “D” to mean “degenerate” star, then:

- **DO** indicates stars which show only He II lines;
- **DB** indicates stars which show only He I lines;
- **DA** indicates stars which show only H lines;
- **DC** indicates stars which show essentially no spectral features (although this could be due to lack of resolution or low S/N, there do appear to be cases where there are no detectable spectral lines !);

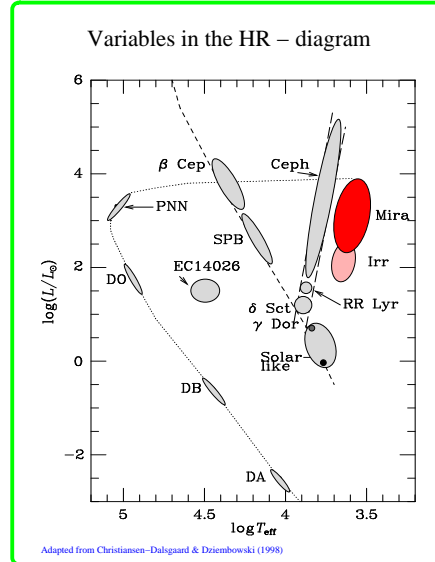


Figure 37: The location of the pulsating white dwarf stars (and planetary nebula nuclei (PNN) in the HR diagram.

- **DZ** indicates stars which show metallic lines;
- **DQ** indicates stars which show atomic or molecular Carbon features.
- Additional letters are used to indicate polarised magnetic stars (P), magnetic stars which show no detectable polarisation (H), and variable stars (V).
- A final number indicates the temperature. This “index” is a number corresponding to $50400/T_{\text{eff}}$. The hottest stars (with $T_{\text{eff}} > 100\,000$ K) have an index of 1.

Combinations of the above can (and do) exist. For example, a white dwarf which had Balmer absorption and weak metal lines would be DAZ and a star with Carbon features and weaker H and He I lines would be DQAB.

Note that because white dwarfs are one of the “end products” of stellar evolution, their atmospheres might have very unusual compositions compared to main sequences stars. Thus the DO, DB and DA stars are not analogues of O, B and A stars. A DA white dwarf, for example, has an atmosphere which is almost pure hydrogen (because of gravitational “settling” of heavier elements) and a DB atmosphere has no Hydrogen at all. Thus, DB and DA spectral types can cover a large *and similar* range of temperatures.

For more details on white dwarf types, see *Publ. astr. Soc. Pacific*, 105, 761 (1993).

3.7.2 DO pulsators (DOV) = PG1159 = GW Vir stars

The DO stars are very hot white dwarfs. A hotter extension to these stars was found with the discovery of the pulsating star PG1159-034 (since re-named GW Vir, although the name “PG1159” persists for both the star and the class of stars).

PG1159 stars show weak He II lines and high ionisation species such as C IV, N V and O VI – often in emission – and are more typically classified DOQZ1 or DZQ1 (see the figure), but the pulsators in the class are still referred to as DOV stars. Their surface temperatures are in excess of 80 000 K.

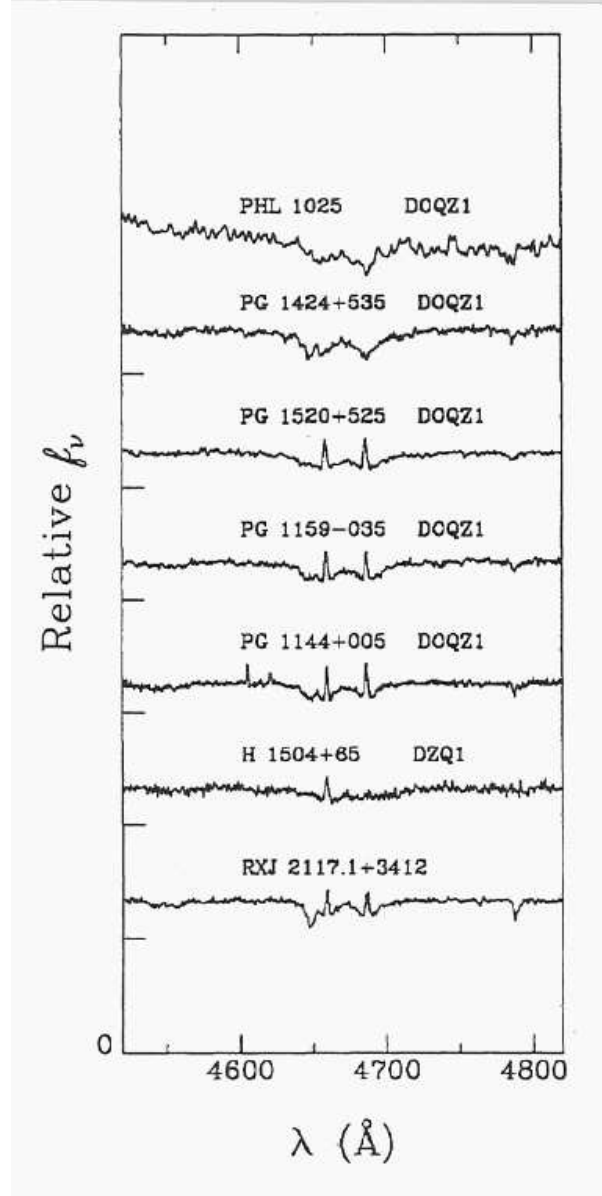


Figure 38: Spectra of some PG 1159 stars. The weak lines are probably blends but include CIV lines around 4650 – 4660 Å and HeII 4686 Å.

Only a small number of PG 1159 stars are known – and an even smaller number (currently only 5) are known to pulsate.

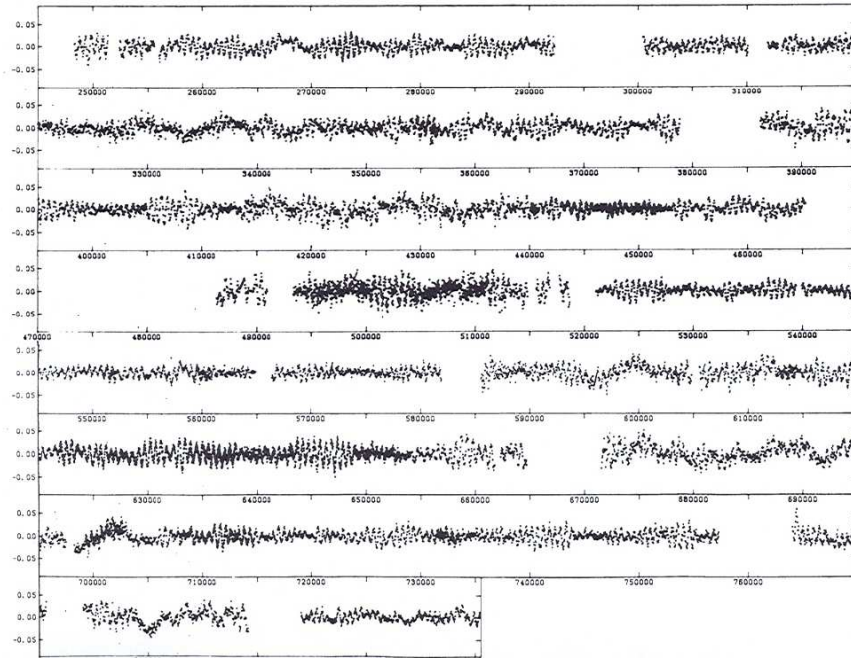


Figure 39: Six days (continuous) “WET” light curve for PG 1159–034.

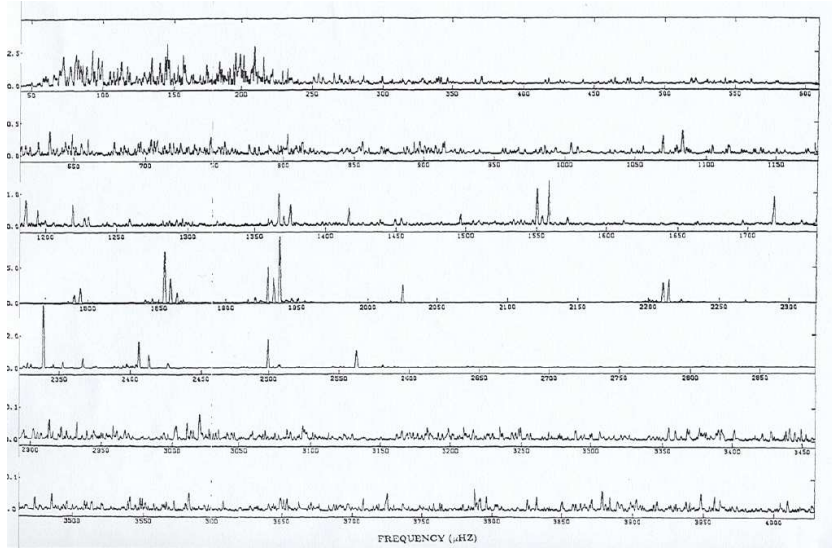


Figure 40: Periodogram for the light curve.

PG 1159–034 itself is probably the best studied of all the DOV stars. The data shown above were part of a Whole Earth Telescope campaign which was able to resolve 125 frequencies (probably the most known for any star other than the Sun). Something like a hundred of these were identified as g-modes and these were used to determine the mass of the star to extraordinary precision ($0.586 M_{\odot}$) and to investigate the stratification of the interior of the star and put upper limits on the strength of the magnetic field (*Astrophysical Journal* 378, 326 (1991)).

3.7.3 DB pulsators (DBV) = V777 Her stars

The DBV stars have temperatures in the range $\sim 22\,000 - 27\,000$ K (though DB stars extend over a much wider temperature range).

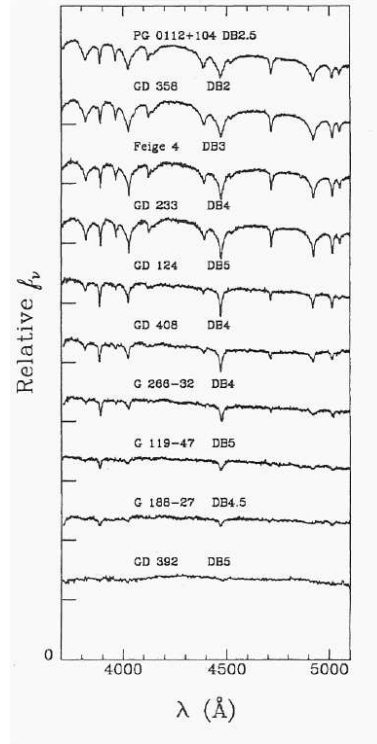


Figure 41: Some DB spectra. The visible lines are HeI (neutral helium) – the strongest are at 3829, 3867, 4026, 4387, 4471, 4713, 4921 Å. Note that as stars in the sequence become cooler, the helium atmosphere no longer shows lines in the visible.

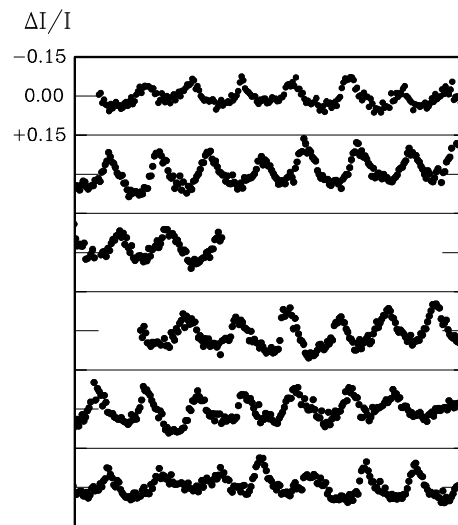


Figure 42: Light curve (about 5 hours) for an EC survey DB star.

3.7.4 DA pulsators (DAV) = ZZ Ceti stars

The DAV stars (commonly called ZZ Ceti stars) have temperatures around 10 000 – 12 000 K, though again DA stars extend over a much wider temperature range (because of the effect of atmospheric composition).

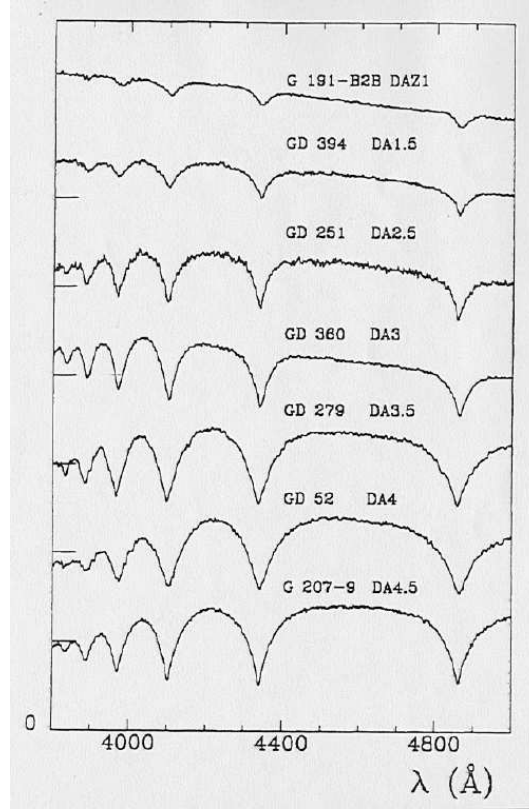


Figure 43: Some DA spectra examples.

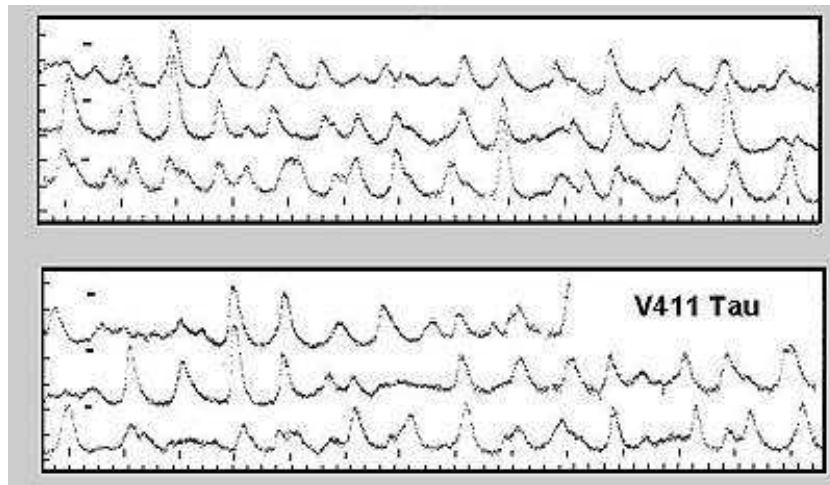


Figure 44: Three nights worth of data for the ZZ Ceti star V411 Tau. Note the obvious complexity of the pulsation.

A recent particularly interesting multi-site campaign (held in 1999 but just recently accepted for publication) investigated the DAV star BPM 37093. It turns out that the observations of this massive ($\sim 1M_{\odot}$) white dwarf provide the first evidence for a phenomenon suggested some 40 years ago by Ed Salpeter – namely that at the enormous pressures inside a cooling white dwarf, the core might begin to crystallise, as indicated in this rather fanciful “artists impression”.

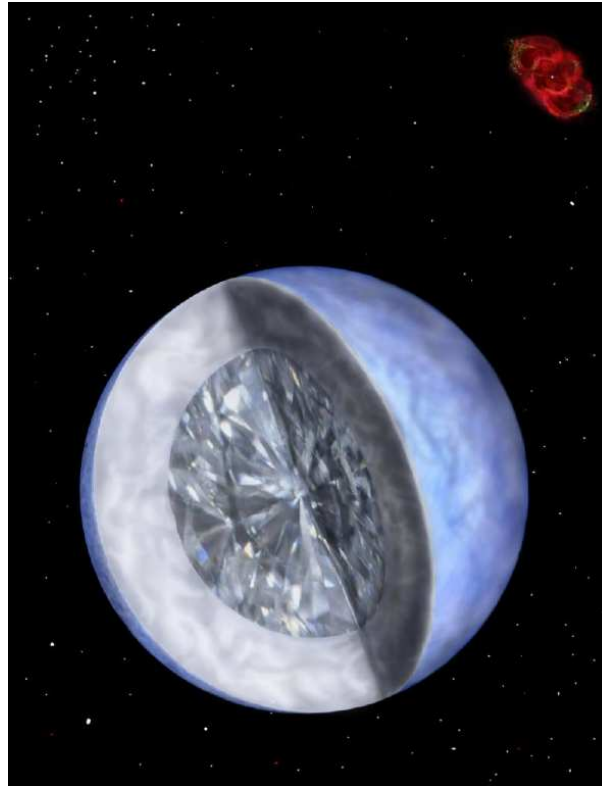


Figure 45: Artists impression of what the interior of BPM37093 probably doesn’t look like !

In fact, what is more likely is that the oxygen in the star would begin to crystallise first and would fall to the centre of the star, followed by the carbon. There might thus be an oxygen/carbon “snow” falling on to an oxygen crystallisation – eventually forming a core of solid oxygen surrounded by a thick shell of crystallised carbon.

The press – particularly in the US – has been keen to tout this as a 10^{33} -carat blue-green diamond, about the size of the Moon and the mass of the Sun, but it is unlikely that the crystal form taken would be that of a diamond. However, it makes a cool picture !

But beyond the pretty picture there is a serious result. We can see a limit to the coolest white dwarfs (~ 3500 K), so if we can measure white dwarf cooling rates, we can determine a lower limit to the age of the Galaxy and, potentially, the Universe. Depending upon how the white dwarf stars cool and crystallise, the release of the latent heat of crystallisation will mean the star will cool more slowly. In this case, the Galaxy might be significantly older than previously thought.

A more detailed, but still general description of white dwarf pulsation can be found in *Sky & Telescope*, April 1992.